A Pictorial Account of the Fundamentals

of Propeller Theory

and its

Application to the Design of a

Pedal Driven Airplane Propeller

by

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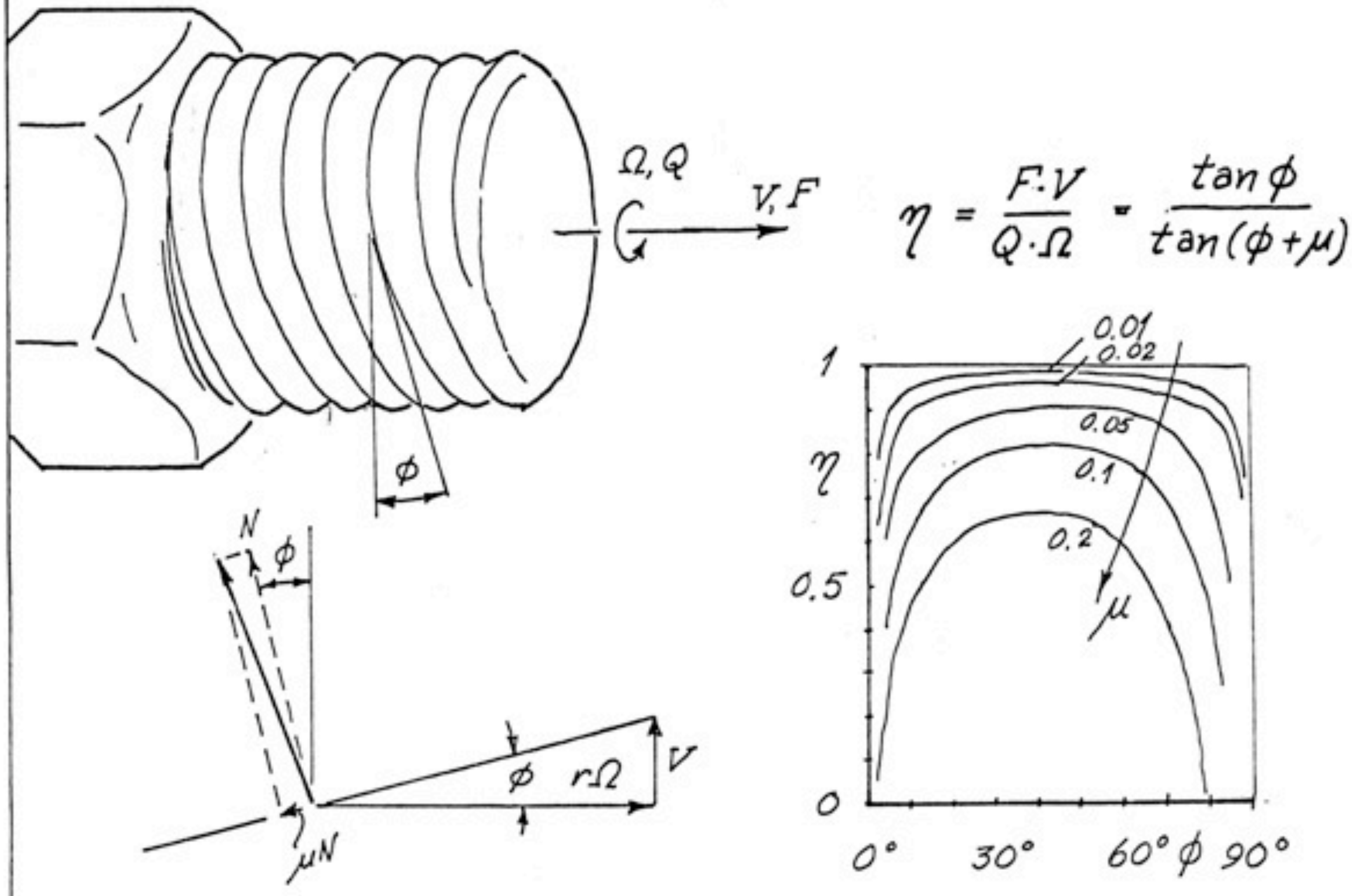
1. The first propellers were expected to screw themselves into the water as a bolt turns itself into a nut. If there is appreciable friction, μ, so that there is a frictional force μN along the threads as well as a normal force N perpendicular to them, the efficiency of the screw - its pull times the linear velocity divided by the torque to turn it times the angular velocity - is found to depend on the thread helix angle φ as well as the angle μ whose tangent equals the coefficient of friction. Screws of highest efficiency have a helix angle φ of 30° to 44° corresponding to pitch-diameter ratios of 1.8 to 3.



EFFICIENCY OF A MACHINE SCREW

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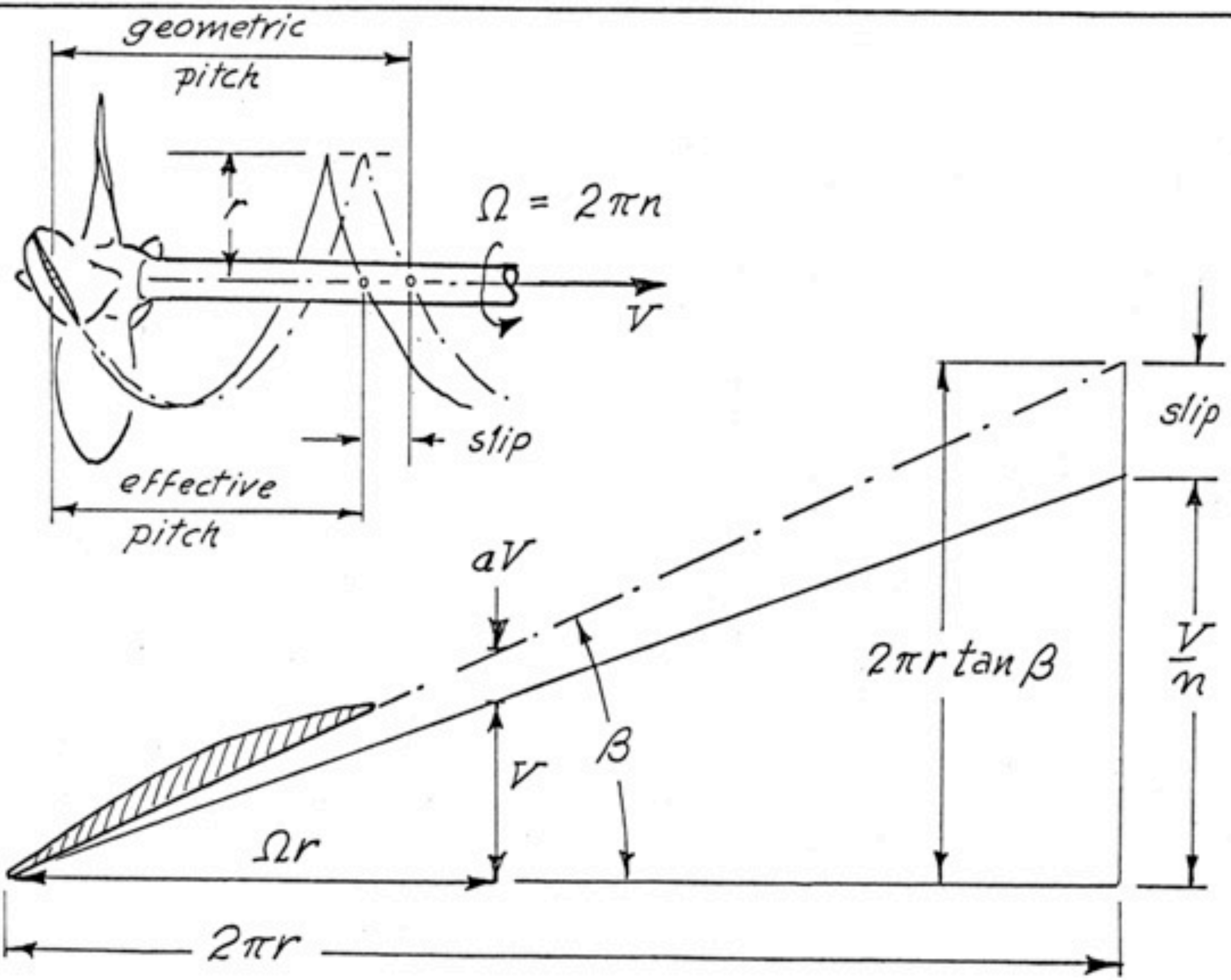
2. As soon as steamboat propellers were built it was discovered that the actual working pitch, the distance travelled per revolution, V/n, was less than the geometric pitch $2 \pi r \tan \beta$, where beta is the geometric angle of the blade section at radius r. The difference between the geometric and the effective pitch is called the slip, and is caused by the motion of the water acted on by the propeller. The "inflow" velocity aV is to the speed of the ship V as the slip is to the effective pitch.



CONCEPT OF SLIP

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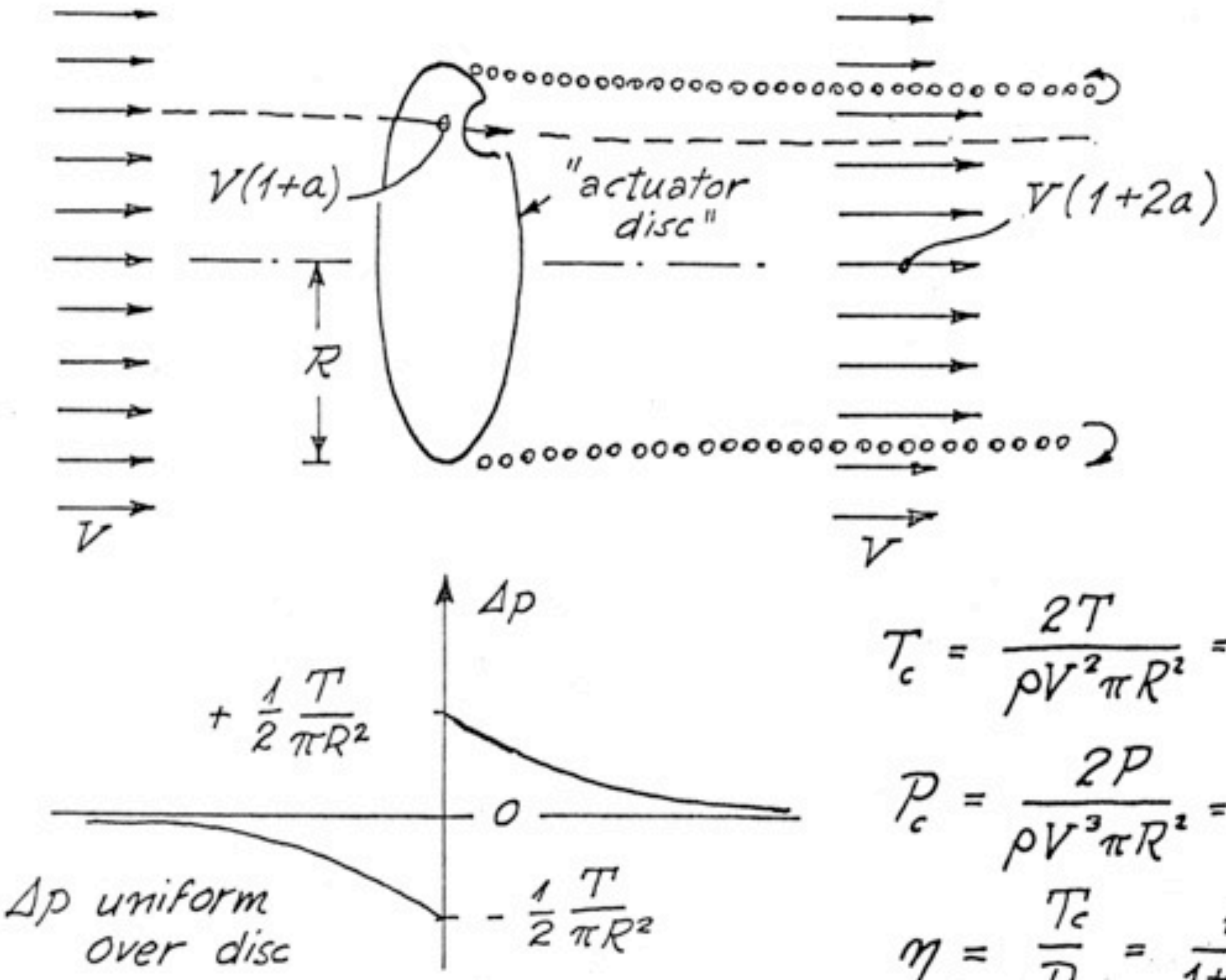
3. In the momentum theory of propellers due to Rankine and Froude, it is shown that the inflow velocity at the propeller, aV, is exactly half the slipstream velocity increase, 2aV, in the "developed" slipstream. The slipstream velocity increase depends on the disc loading, $T/ \pi R^2$, the fluid density p and the ship speed V. The shaft power P is the sum of the output (the thrust times the ship speed) and the losses (the increase of slipstream kinetic energy per unit time). The thrust and shaft power are conveniently expressed by dimensionless coefficients Tc and Pc which can be written in terms of the axial inflow velocity ratio a. The theory shows that excessive values of a give rise to poor efficiency, but gives no information about how much thrust a propeller of given geometry can be expected to produce. A satisfactory propeller theory awaited the invention of the airplane and an appropriate theory of the lift of wings.



$$T_c = \frac{2T}{\rho V^2 \pi R^2} = 4a(1+a)$$

$$P_c = \frac{2P}{\rho V^3 \pi R^2} = 4a(1+a)^2$$

$$\eta = \frac{T_c}{P_c} = \frac{1}{1+a}$$

(1865) (1889)

RANKINE - FROUDE MOMENTUM THEORY

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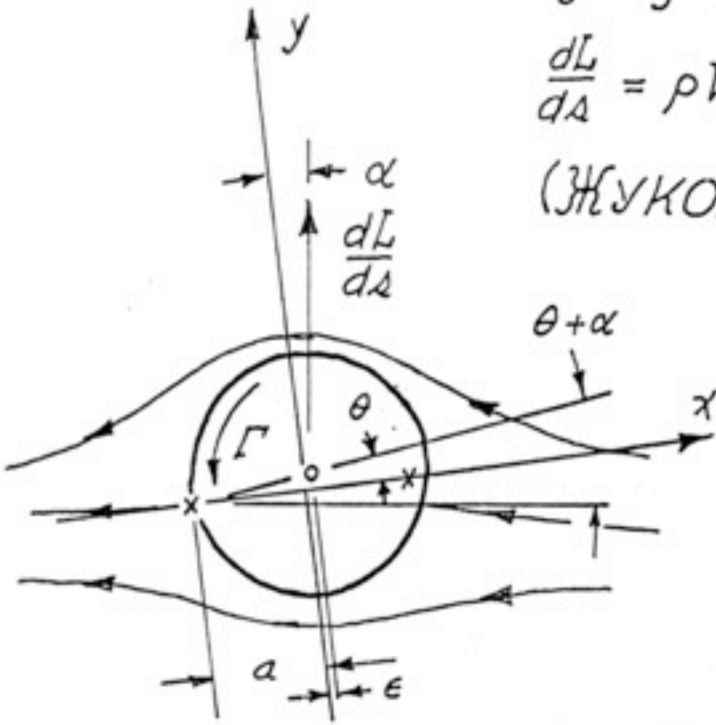
4. A correct theory of the lift acting on wing and propeller blade sections was not published until 1911 by Kutta (in Germany) and Joukowsky (in Russia). In Joukowsky's formulation the flow about a circle is conformally transformed into the flow about an airfoil. The lift per unit span dL/ds about either the circle or the airfoil is given by $\varrho V\Gamma$ where Γ is the strength of the vortex added to the flow at the center of the circle Γ is chosen to make the region at $x = -a$, $y = 0$ a stagnation point, which causes the flow to be aligned with the trailing edge of the Joukowsky airfoil in the ξ, η plane. The lift coefficient of the Joukowsky airfoil (and all thin airfoils) increases at the rate of 2π per radian (0.109662 per degree) angle of attack in the unstalled range 5° of φ angle corresponds to a circular arc camber of 4.37%, or a lift coefficient of 0.54831 at zero angle of attack.

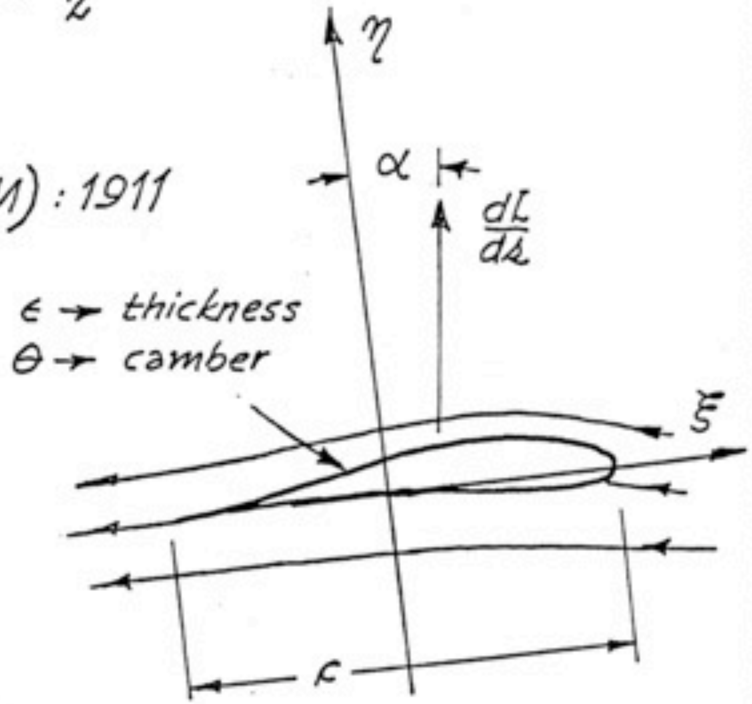
$$z = x + iy$$

$$\zeta = \xi + i\eta = z + \frac{a^2}{z}$$

$$\frac{dL}{ds} = \rho V \Gamma$$

(ЖУКОВСКИЙ) : 1911





$$c_l = \frac{dL/ds}{\frac{\rho}{2} V^2 c} \cong 2\pi \sin(\alpha + \theta)$$

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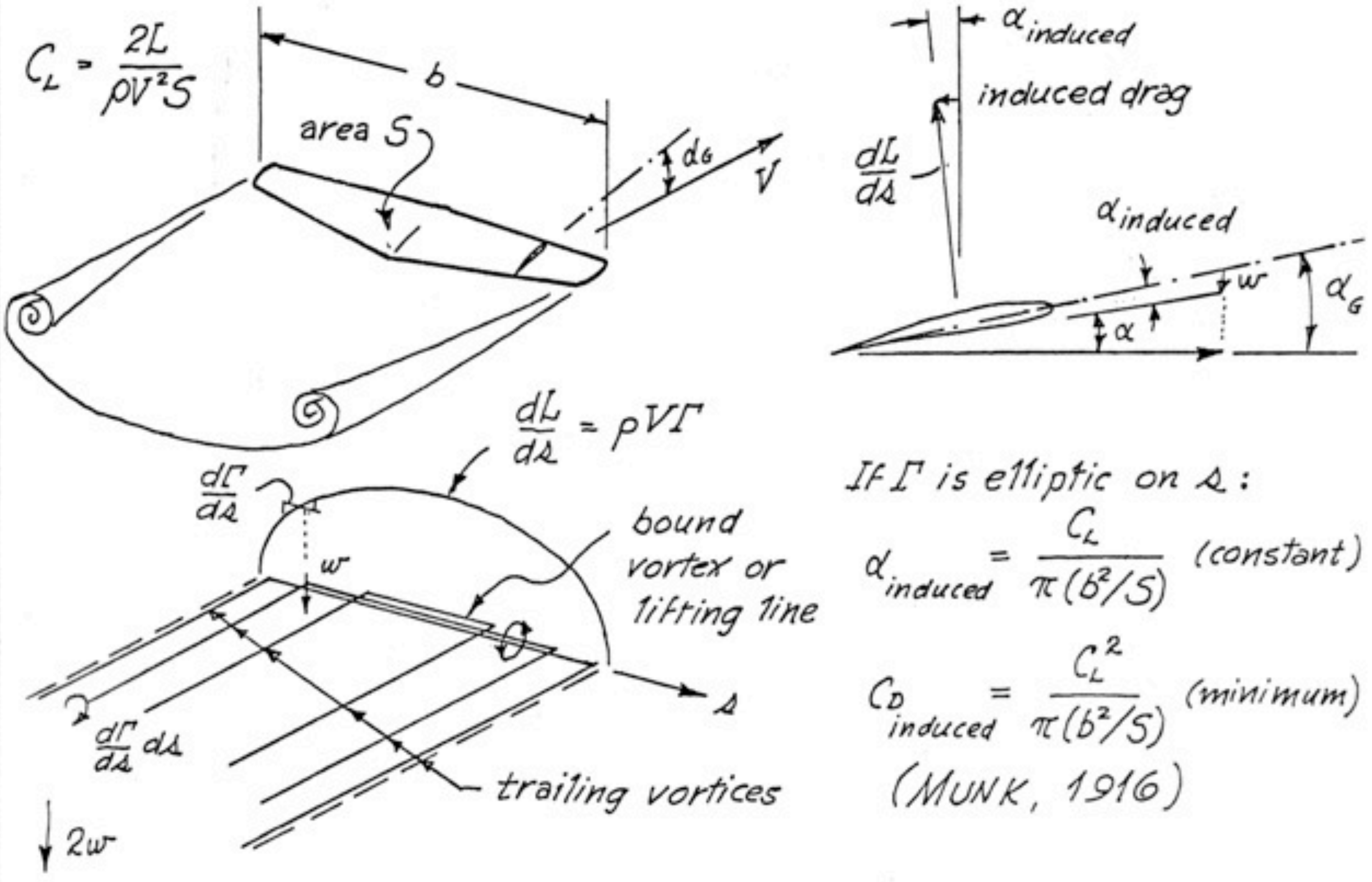
JOUKOWSKY'S VORTEX THEORY OF AIRFOILS

5. A correct theory of wings must account for the flow about the wing tips. The first one to do so was developed at Goettingen during the first world war. The theory accounts for a trailing vortex system. Each elementary trailing vortex filament in the trailing vortex sheet has a strength proportional to the spanwise gradient of the bound vortex, which corresponds to the lifting vortex of Joukowsky's airfoil theory. Max Munk showed (ca. 1916) that if the lift (or the bound circulation) varies elliptically in the spanwise direction, the vortex sheet will convect itself downward with a uniform velocity 2w far behind the wing, and that the air meeting the wing sections will have a uniform downward velocity half as great. The wing sections will have to be set at a geometric angle α_G which is larger than the section angle of attack α by the induced angle of attack w/V, and. there will be an induced drag caused by rotating the lift vectors aft 'through the induced angle. Constant downwash w and elliptic span loading correspond to minimum induced drag. The general case of non-elliptic span loading and non-minimum induced drag was not solved satisfactorily until 1925 by Glauert.



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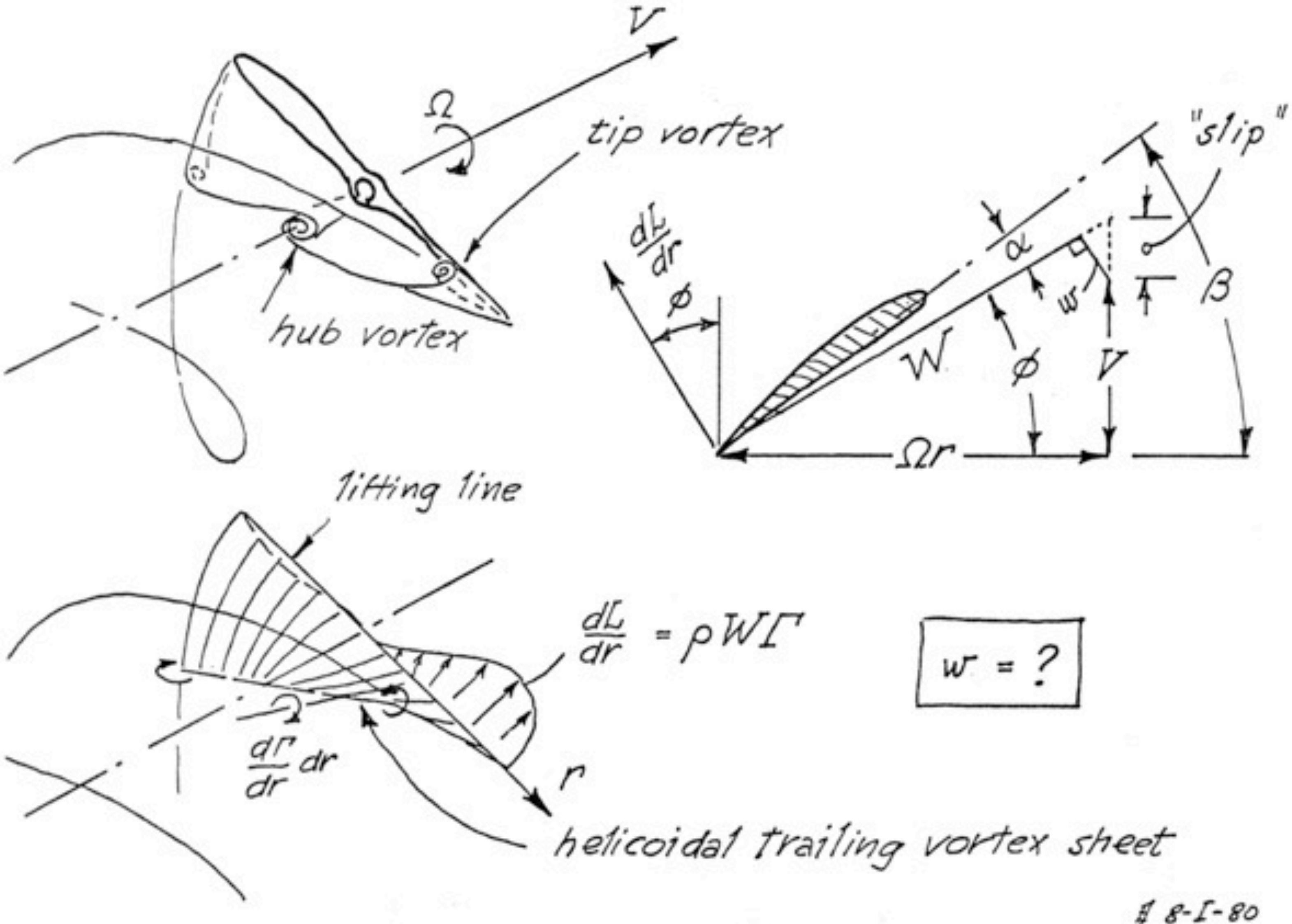
GOETTINGEN VORTEX THEORY OF WINGS

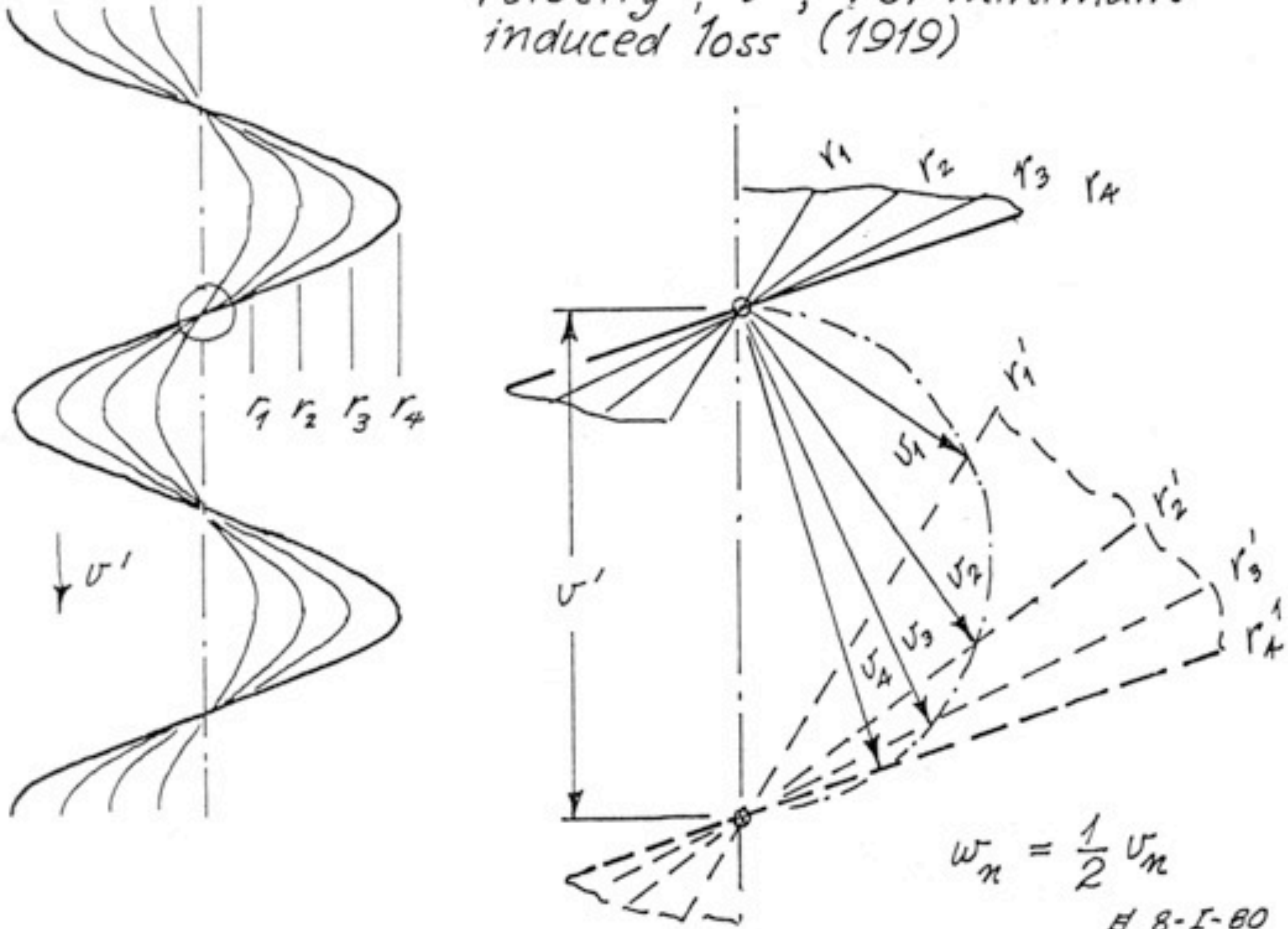
6. The vortex theory of propellers is like the vortex theory of wings except that the trailing vortex sheets are helicoidal surfaces. The induced velocity w can be expected to be perpendicular to the surface of the helicoidal sheet, and is related to the "slip" of the Rankine - Froude momentum theory as shown by the velocity diagram in the upper right corner. At first no convenient mathematical process could be discovered to relate w to the radial gradient of lift, dL/dr, on all the blade elements.



GOETTINGEN VORTEX THEORY OF PROPELLERS

7. Albert Betz, of the Goettingen group, discovered the helicoidal vortex sheet motion for minimum induced loss about 1919. A helical vortex filament at radius r_1 moves perpendicular to itself with velocity v_1, at radius r_2 with velocity v_2, etc., so that each filament has such a combination of axial motion and rotational motion that all appear to have a radially constant "displacement" velocity v'. This is the propeller counterpart of the spanwise uniform downwash velocity behind a wing of minimum induced drag. The induced velocity at any radius is half the sheet velocity at the same radius.

Betz: radially constant "displacement velocity", v', for minimum induced loss (1919)



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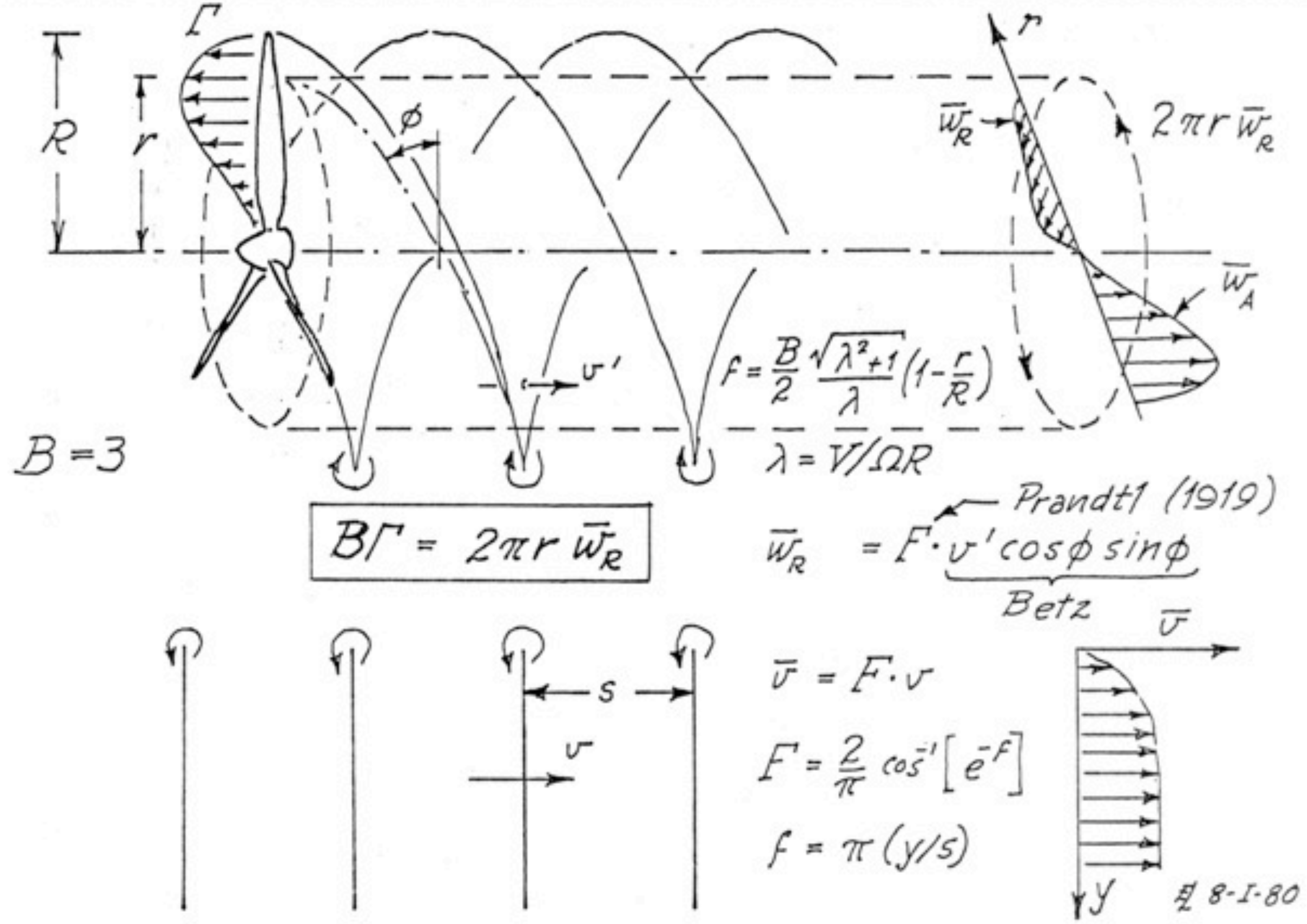
HELICOIDAL VORTEX SHEET MOTION FOR MINIMUM LOSS

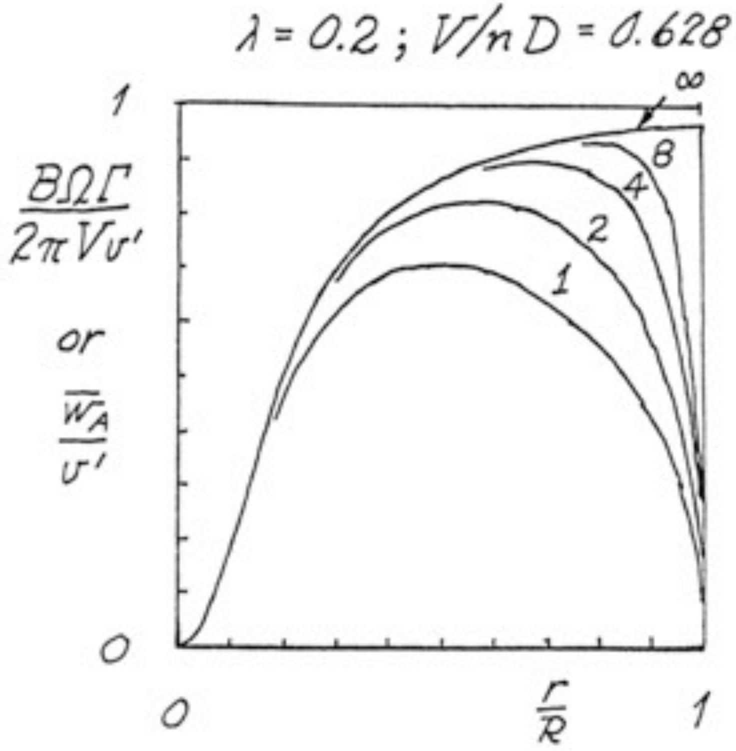
8. Prandtl, the head of the Goettingen group, calculated the radial variation of the lifting vortex strength Γ, corresponding to minimum induced loss propeller operation, by setting the total bound "circulation," $B\Gamma$, at the radius r equal to the circulation about a tube of developed slipstream of the same radius, $2 \pi r w_R$. The average rotational velocity w_R at this radius is a fraction F of the sheet rotational velocity which depends on the number of blades B, the sheet helix angle as specified by the advance ratio $\lambda = V/\Omega R$, and the distance from the sheet edges as given by (1-r/R). The value of the fraction F was taken by analogy from the known solution for the flow about the edges of an infinite array of semi infinite plates moving perpendicular to themselves. Goldstein, in 1929, was to confirm Prandtl's approximate calculation for the practically important case of small tip helix angles and small vortex sheet edge spacing compared to the tip radius.

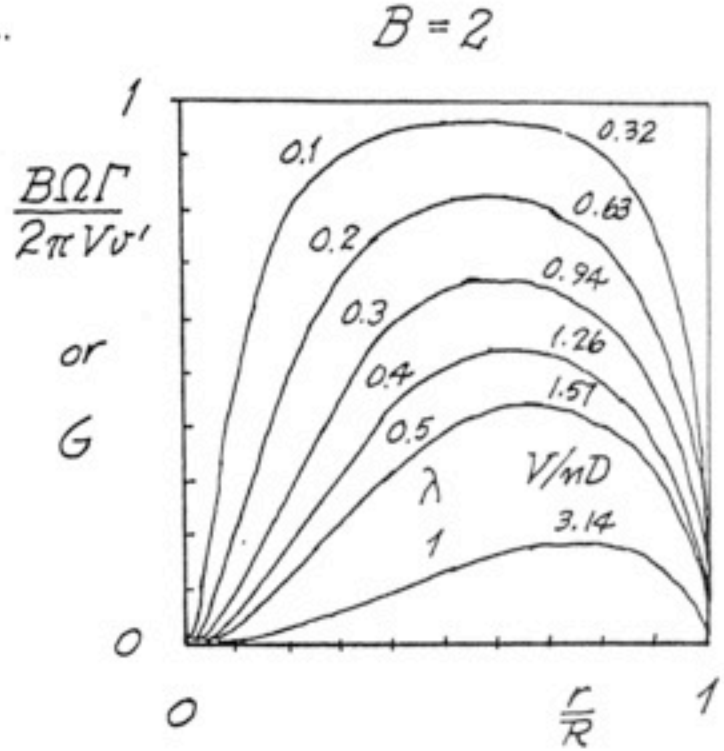


CALCULATION OF THE MINIMUM LOSS CIRCULATION FUNCTION

9. Radial circulation distributions according to Prandtl and Betz for minimum induced loss propeller operation. They are the propeller theory counterpart of elliptic circulation distribution in wing theory.

The dimensionless circulation parameter $B\,\Omega\,\Gamma/2\,\pi\,rVv'$ is numerically identical to the ratio of the average axial velocity increase w_A in the developed slipstream divided by the displacement velocity v', and is desirably equal to 1 at all radii. Flat pitch ($\lambda = 0.1$) propellers with 2 or more blades would be desirable for minimum induced loss, but might be undesirable for other reasons.





Effect of number of blades, B

Effect of advance ratio, λ

$$B\Omega\Gamma/2\pi V v' = F x^2/x^2+1 \quad ; \quad x = \Omega r/V = \frac{r/R}{\lambda}$$

(Betz u. Prandtl)

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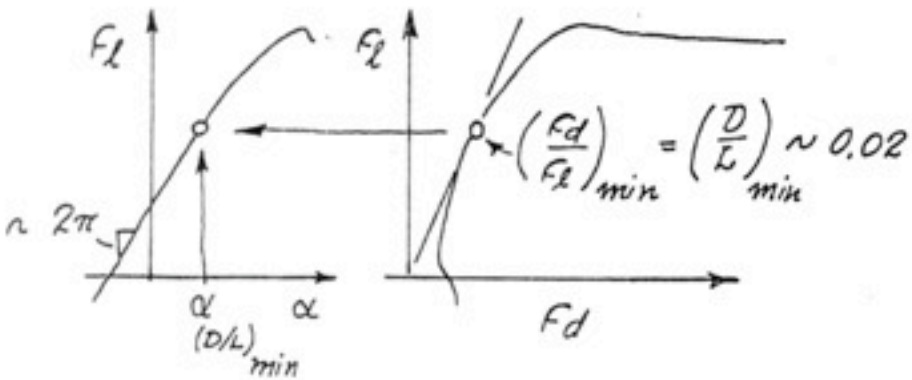
MINIMUM INDUCED LOSS CIRCULATION DISTRIBUTIONS

10. The design of a minimum induced loss propeller is very straightforward once the displacement velocity, v' is determined. The blade angle of attack and lift coefficient at each radius is chosen to minimize the profile drag to lift ratio. The blade chord and angle at each radius depends on the displacement velocity as given by the formulas and the trigonometry of the velocity diagram.

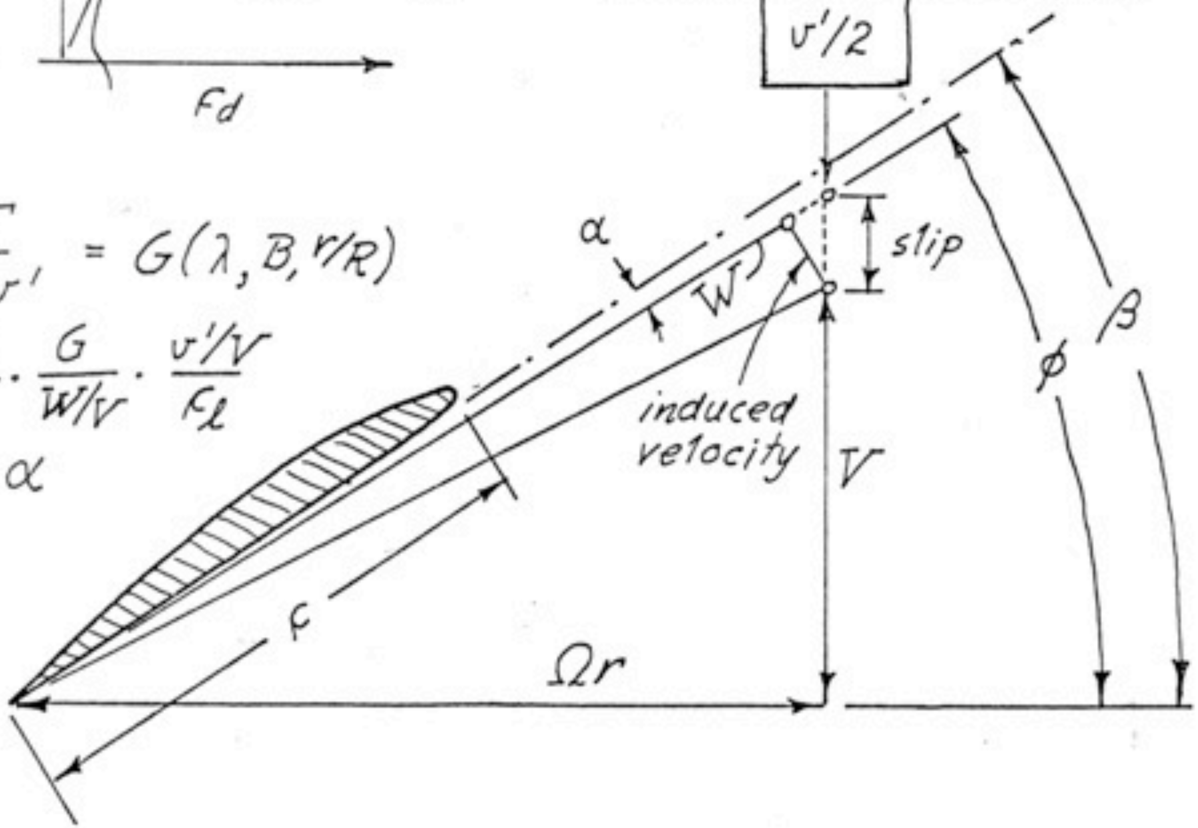
airfoil properties



$$G = \frac{B\Omega\Gamma}{2\pi V v'} = G(\lambda, B, r/R)$$

$$\frac{c}{R} = \frac{4\pi\lambda}{B}\cdot\frac{G}{W/V}\cdot\frac{v'/V}{c_l}$$

$$\beta = \phi + \alpha$$



MINIMUM INDUCED LOSS PROPELLER DESIGN

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11. Professor Larrabee's method for calculating the displacement velocity ratio, $v'/V = \zeta$, for specified power P or thrust T. Four integrals I_1, I_2, J_1, and J_2 must be numerically evaluated which depend only on the number of blades B, the advance ratio λ, and the radial variation of blade element D/L ratio. The formulas for C in terms of the power coefficient Pc or thrust coefficient Tc are the propeller counterpart of the formula $\alpha_i = C_L/w(b2/S)$ for the spanwise constant induced angle of attack of an elliptically loaded wing.

SPECIFIED POWER:

$$P_c = \frac{2P}{\rho V^3 \pi R^2}$$

SPECIFIED THRUST:

$$T_c = \frac{2T}{\rho V^2 \pi R^2}$$

$$\zeta = \frac{J_1}{2J_2}\left[\sqrt{1+\frac{4P_c J_2}{J_1^2}} - 1\right] = \frac{v'}{V} = \frac{I_1}{2I_2}\left[1-\sqrt{1-\frac{4T_c I_2}{I_1^2}}\right]$$

$$T_c = I_1\zeta - I_2\zeta^2 \qquad\qquad P_c = J_1\zeta + J_2\zeta^2$$

$$\eta = \frac{T_c}{P_c}$$

$$I_1 = 4\int_0^1 \xi G\left(1-\frac{D/L}{x}\right)d\xi \qquad\qquad J_1 = 4\int_0^1 \xi G\left(1+\left(\tfrac{D}{L}\right)x\right)d\xi$$

$$I_2 = 2\int_0^1 \frac{\xi G\left(1-\frac{D/L}{x}\right)d\xi}{x^2+1} \qquad\qquad J_2 = 2\int_0^1 \frac{\xi G\left(1+\left(\tfrac{D}{L}\right)x\right)\left(x^2\right)d\xi}{x^2+1}$$

$$\xi \equiv r/R \qquad\qquad x = \Omega r/V = \xi/\lambda$$

CALCULATION OF THE DISPLACEMENT VELOCITY, v'

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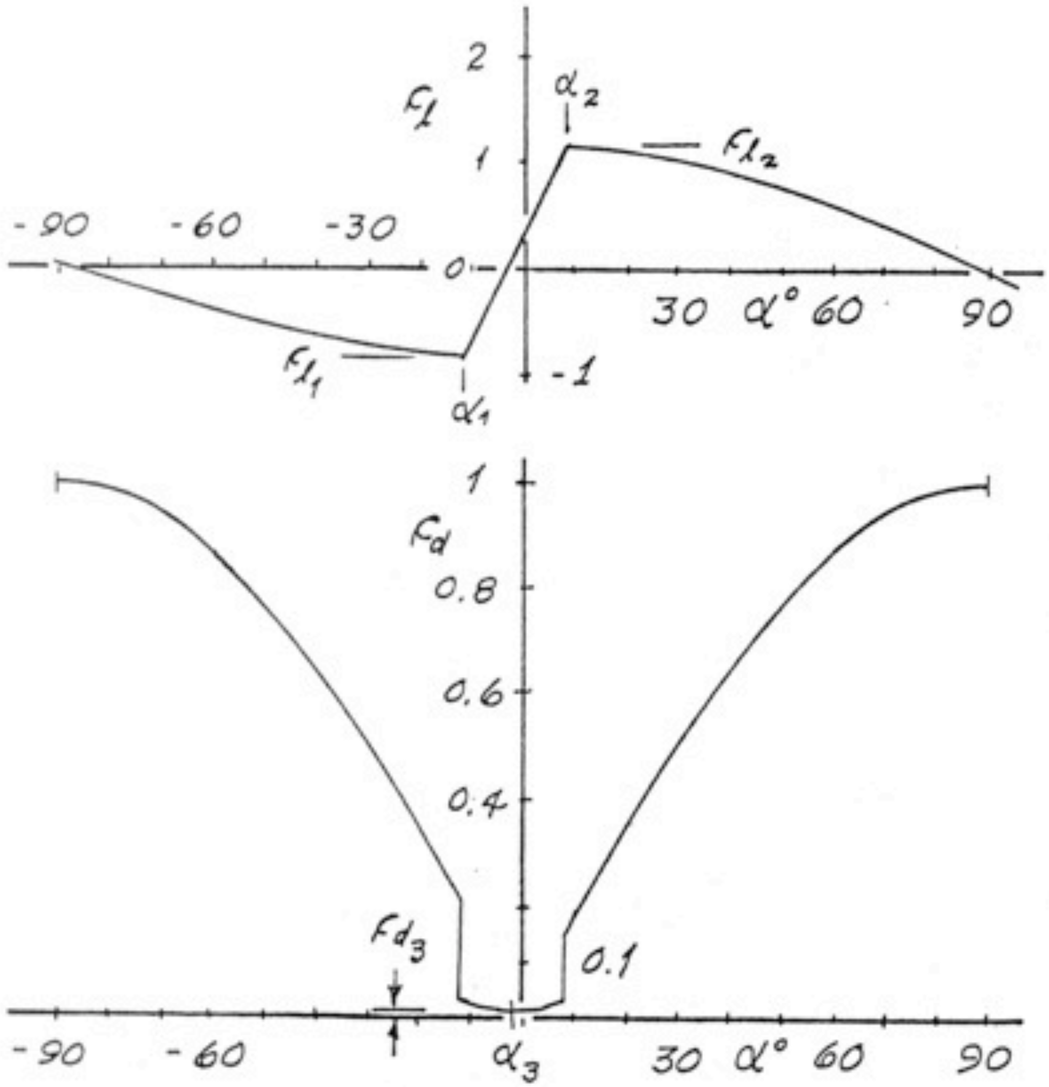
12. A complete propeller analysis requires the specification of airfoil section properties from alpha equals -90° to +90°. This is the seven parameter analytic airfoil section property specification used in M.I.T.'s HELICE propeller program. The particular values plotted correspond to the default entries in the program.

HELICE AIRFOIL SECTION PROPERTIES



Stall routine:

$\alpha \leqq \alpha_1, \alpha \geqq \alpha_2$:

$$c_l = \frac{c_{l_1}}{\cos\alpha_1} \cos\alpha$$

$$c_l = \frac{c_{l_2}}{\cos\alpha_2} \cos\alpha$$

$$c_d = |\sin\alpha|$$

Default values

$c_{l_1} = -0.8$

$\alpha_1 = -12.0°$

$c_{l_2} = +1.2$

$\alpha_2 = +8.0°$

$c_{d_3} = +0.008$

$\alpha_3 = -2.0°$

$dc_d/d(\alpha^2) = 0.000\,25$ $(deg)^{-2}$

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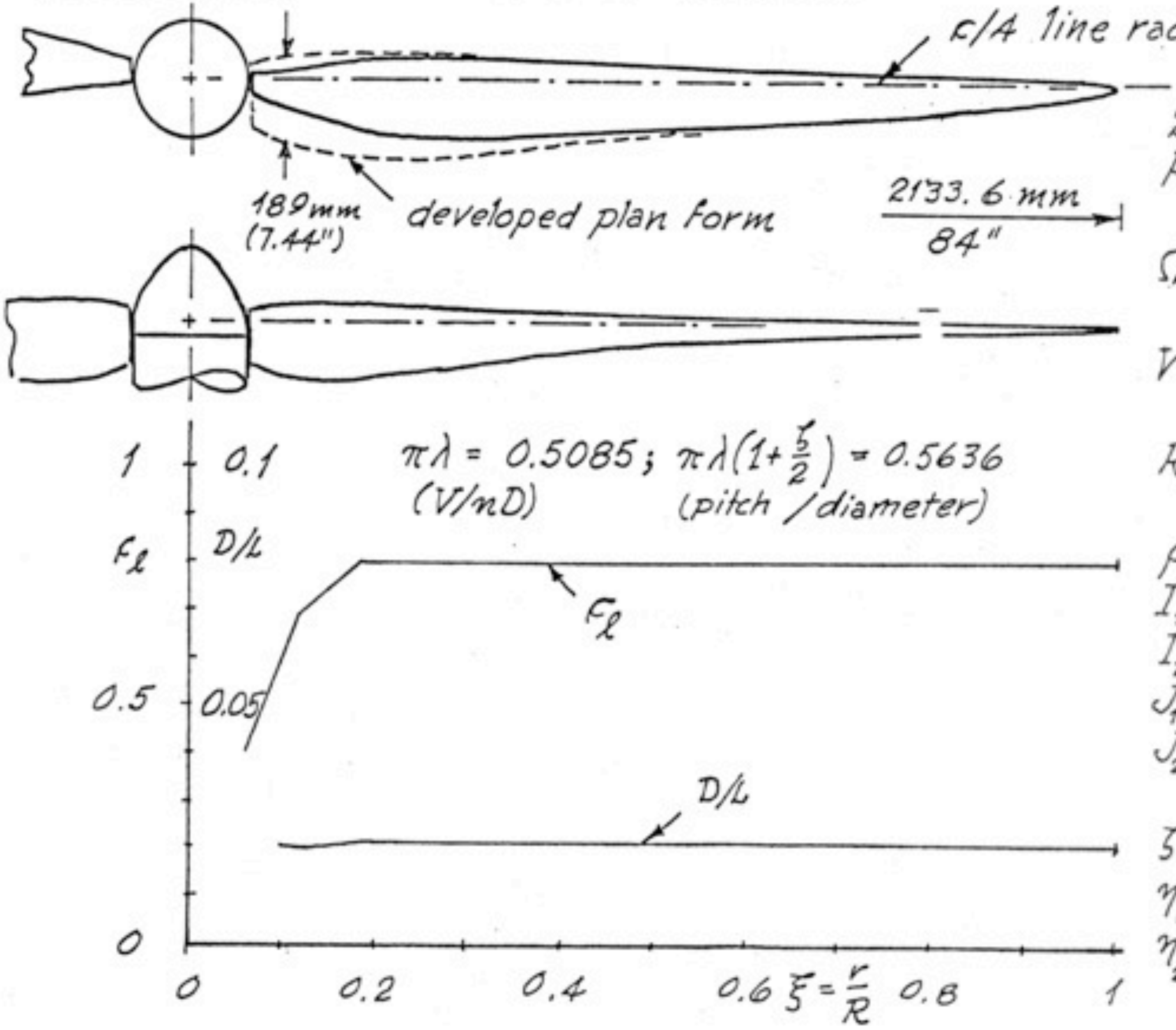
A

13. Redesign of the Chrysalis human powered airplane propeller with the HELICE program. The two bladed propeller is required to absorb 373 W (0.5 hp) at a shaft speed of 14.137 rad/s (2.25 rps) and a flight speed of 4.877 m/s (16 fps) with a radius of 2.1336 m (7 ft) in air of standard sea level density, 1.225 Kg/m^3. The design lift coefficient is radially constant at a value of 0.8, except near the hub where it is purposely reduced so that the propeller stub spars will be within airfoil contour. The drawing shows the geometry of the propeller as calculated by the program which also yields the integrals I_1, I_2, J_1, and J_2 and two values for the design point efficiency, as explained in Professor Larrabee's paper, "Propeller Design for Motor soarers," NASA Conference proceedings 2085, pps 285-303. The design program incorporates the effect of blade element "friction" in the form of the radial variation of the blade element drag to lift ratio.

CHRYSALIS PROPELLER DESIGN



2 blades

$P = 373\ W$ (0.5hp)

$\Omega = 14.137/s$ (2.25 rps)

$V = 4.877\ m/s$ (16 ftps)

$R = 2.1336\ m$ (84")

$\rho = 1.225\ kg/m^3$

$I_1 = 1.4123$

$I_2 = 0.06312$

$J_1 = 1.5409$

$J_2 = 0.7028$

$\zeta = 0.2167$

$\eta_1 = 0.8260$

$\eta_2 = 0.8326$

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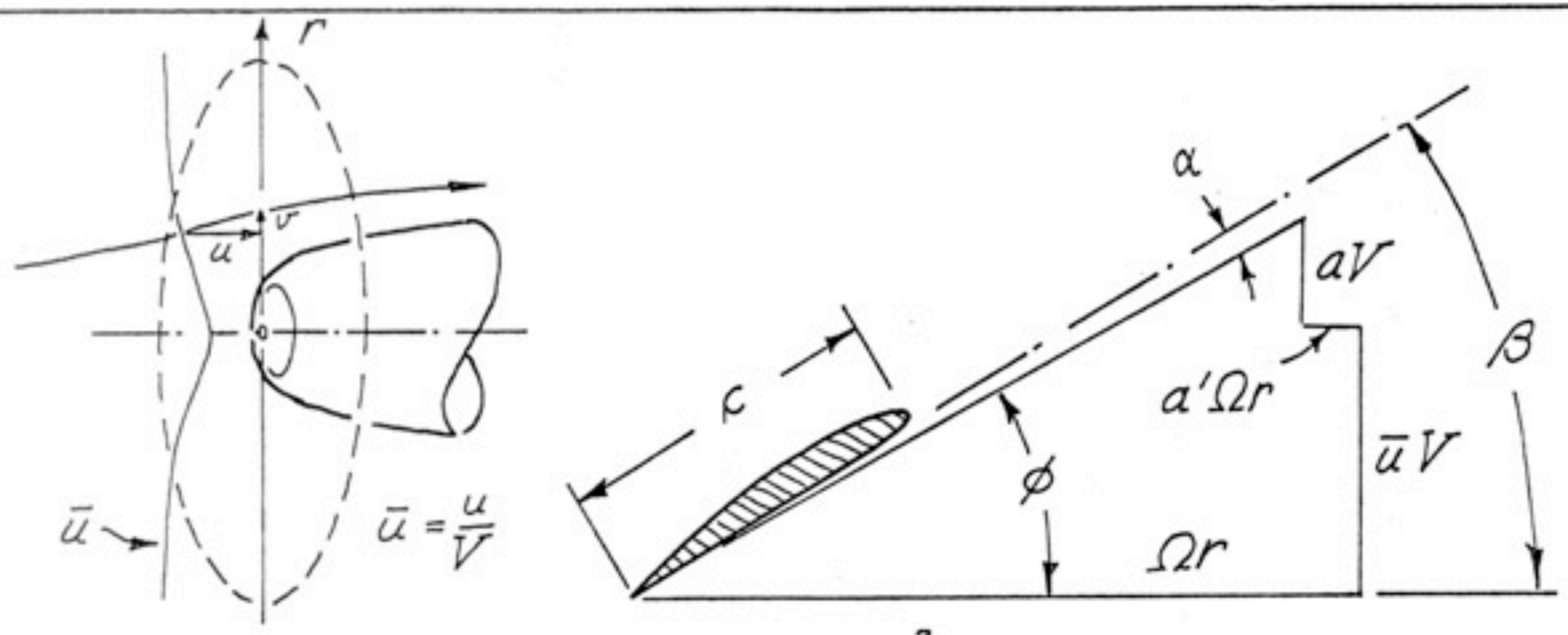
B

14. In order to determine the performance of a minimum induced loss propeller away from its design point (or to determine the performance of a propeller of arbitrary geometry), it is necessary to calculate the axial aV and rotational $a'\Omega r$ components of the induced velocity at every radius. Here is a radially graded momentum theory which calculates these velocities by relating the slipstream momentum changes to the thrust (C_y) and torque (C_x) components of the blade element airload. The results are made to be consistent with the performance and geometry of a minimum induced loss propeller at its design point by introduction of Prandtl's slipstream velocity averaging factor F. Distortion of the propeller flow field due to fuselage interference can be accounted for also.



$$\frac{dT_c}{dr} = 2\pi\rho r V^2(\bar{u}+a)2Fa = \frac{1}{2}\rho V^2\frac{(\bar{u}+a)^2}{\sin^2\phi}Bc\left(c_l\cos\phi - c_d\sin\phi\right)$$

$$\frac{1}{r}\frac{dQ}{dr} = 2\pi\rho r V\Omega r(\bar{u}+a)2Fa' = \frac{1}{2}\rho V^2\frac{(\bar{u}+a)^2}{\sin^2\phi}Bc\left(c_l\sin\phi + c_d\cos\phi\right)$$

$$\frac{a}{\bar{u}+a} = \frac{1}{4F}\frac{\sigma C_y}{\sin^2\phi}$$

$$\frac{a'}{1-a'} = \frac{1}{4F}\frac{\sigma C_x}{\sin\phi\cos\phi}$$

$$\sigma \equiv Bc/2\pi r$$

$$C_y = c_l\cos\phi - c_d\sin\phi$$

$$C_x = c_l\sin\phi + c_d\cos\phi$$

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15. MINIMUM-INDUCED-LOSS-CONSISTENT RADIALLY GRADED MOMENTUM THEORY

Another aspect of the HELICE program is the implementation of the radially graded momentum theory outlined in the previous figure. Here it is used to determine the performance of the Chrysalis propeller all the way from blade stall at advance ratios less than 0.13 to windmilling at an advance ratio of 0.28. HELICE permits rapid design of propellers of highest efficiency matched to design point operation and rapid exploration of off design performance. The propeller coefficients given here, $C_T = T/\rho n^2 D^4$ and $C_p = T/\rho n^3 D^5$ are standard propeller coefficients based on n, the shaft speed in revolutions/s. They are plotted versus the effective pitch/diameter ratio $J = V/nD = \pi\lambda$, where D is the propeller diameter. Radial distributions of thrust coefficient and lift coefficient are also given.